St Teresa's Catholic Academy

'Our children are receptive, inquisitive learners who, through our Gospel values, have a unique sense of the world.'

Calculation Policy for Mathematics

About our Calculation Policy

The following calculation policy has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics. It is also designed to give pupils a consistent and smooth progression of learning in calculations across their primary mathematics learning. Please note the early learning in number and calculation in our Preschool and Foundation Stage follows the 'White Rose Scheme of Learning' document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage. This calculation policy has been divided into concrete, pictorial and abstract (CPA) sections throughout. This is to ensure that CPA is use throughout the curriculum to support the children's learning.

Age Stage Expectations

This calculation policy is designed for a smooth transition from one method to the next, meaning that there is no definitive stage that each pupil should be at in relation to their year group. It is vital that pupils are taught according to the stage that they are currently working at, pupils will not be moved on to the next stage until they are secure in their understanding of the stage that is appropriate for their ability. In the National Curriculum 2014, it is expected that the majority of pupils will move through the programmes of study at 'broadly the same pace', with pupils who grasp concepts rapidly being challenged through rich and sophisticated tasks and challenges that deepen their understanding of concepts rather than moving them on to a new concept. Lower attaining children should be supported primarily through scaffolding and support through quality first teaching. This may also be supplemented with interventions.

Providing Context for Calculation: Problem Solving

It is imperative that any type of reasoning calculation is given a real-life context to help build children's understanding of the purpose of the calculation, and to help them recognise when to use certain operations and methods when faced with problems. This ensures that we meet the problem solving and reasoning aims set out in the National Curriculum 2014, whilst allowing the children to have a deeper understanding of the problems they are solving.

Addition						
EYFS: Develop a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers.	**Year 1:** Read, write and interpret mathematical statements involving addition (+) and equals (=) signs. Represent and use number bonds within 20. Add one-digit and two-digit numbers to 20, including zero. Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 7 = ? + 3.	**Year 2:** Recall and use addition facts to 20 fluently, and derive and use related facts up to 100. Add numbers using concrete, pictorial and abstract representations, including: -A two-digit number and ones. -A two-digit number and tens. -Two two-digit numbers. -Adding three one-digit numbers. To recognise and understand the inverse.	**Year 3:** Add numbers mentally, including: -A three-digit. number and ones -A three-digit. number and tens -A three-digit number and hundreds. Add numbers with up to three digits, using formal written methods of columnar addition.	**Year 4:** Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate.	**Year 5:** Add whole numbers with more than 4 digits, including using formal written methods (columnar addition).	**Year 6:** Add whole numbers with more than 4 digits, including using formal written methods (columnar addition).

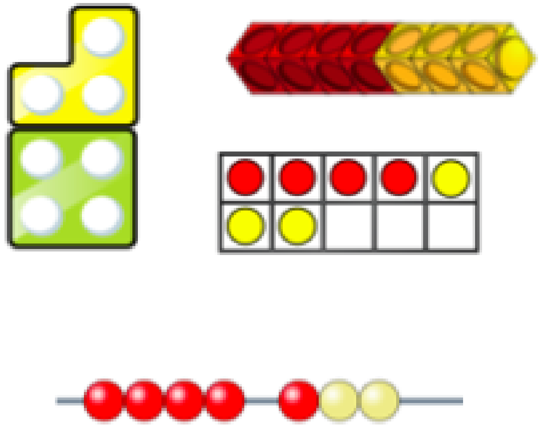
Key Vocabulary:	**STEM Sentences:**
Addend – A number to be added to another. **Aggregation** – Combining two or more quantities or measures to find a total. **Augmentation** – Increasing a quantity or measure by another quantity. **Commutative** – Numbers can be added in any order. **Complement** – In addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000. **Exchange** – Change a number or expression for another of an equal value. **Partitioning** – Splitting a number into its component parts. **Subitise** – Instantly, recognising the number of objects in a small group without needing to count. **Subtrahend** – A number to be subtracted from another. **Sum** – The result of an addition. **Total** – The aggregate or the sum found by addition.	'One more than ____ is ____.' 'I know that ____ add ____ is equal to ____ ; therefore, ____ add ____ is equal to ____.' '____ add ____ is equal to ____.' 'We line up the ones; ____ ones add ____ ones equals ____ ones. We line up the tens; ____ tens add ____ tens equals ____ tens.' Etc. 'We know there are ten hundreds in one thousand, so ____ hundred add ____ hundred is equal to ____ thousand ____ hundred.'

Stage 1

Add 1-digit numbers within 10

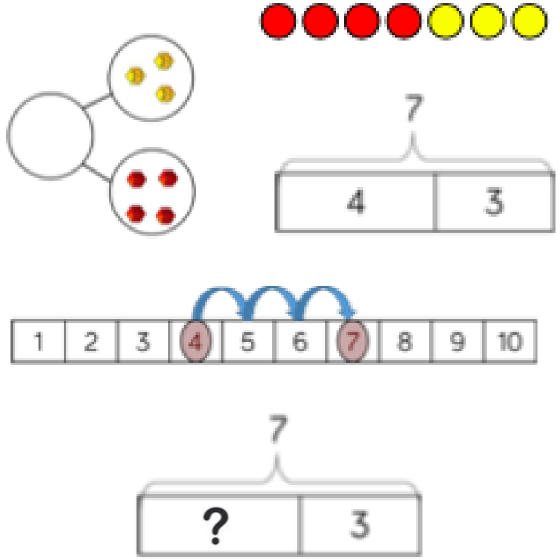
Example question: 4 + 3 = 7

Concrete:



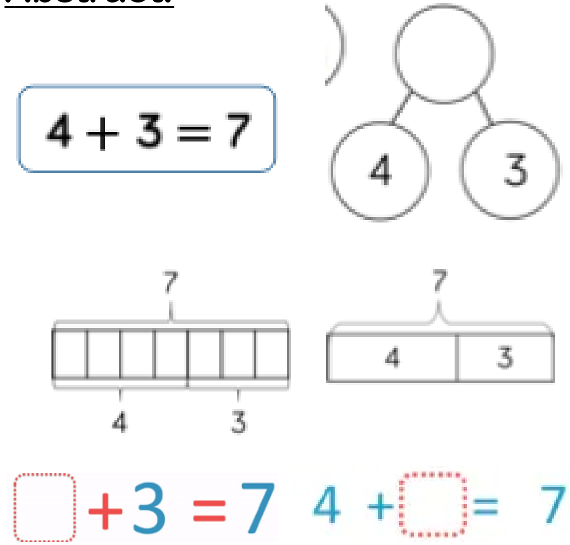
When children are adding numbers to 10, they can explore both aggregation (the formation of a number of things into one group) and augmentation (making an amount greater). Children should be given concrete representations to explore number, both independently and during adult led activities and teaching. Tell a story: there are 3 yellow cars and then 4 red cars join them. There are 8 cars altogether. 3 + 4 = 7.

Pictorial:



After using concrete resources, children should be introduced to pictorial representations of the same calculation that they have been exploring with the concrete resources. Following this, they can then explore new calculations using pictorial representations. Ensure that bar model and part-whole models are rotated and represented at different angles.

Abstract:



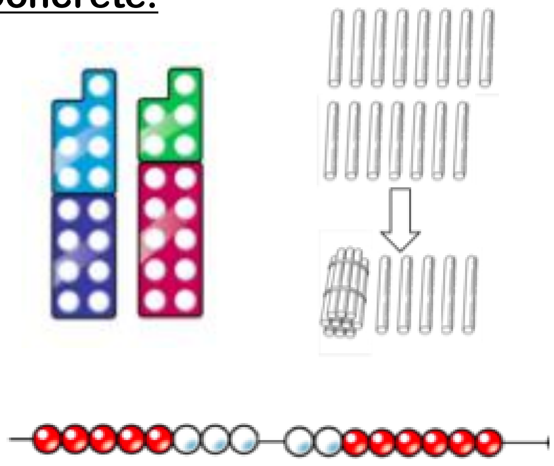
Once the children have understood addition using concrete and pictorial representations, they should then be introduced to the abstract. At this stage the abstract representations should be used in conjunction with the concrete and pictorial – not on their own. From the start, the equals sign should be presented to children on both sides of the calculations.

Stage 2

Add 1 and 2-digit numbers within 20

Example question: 8 + 7 = 15

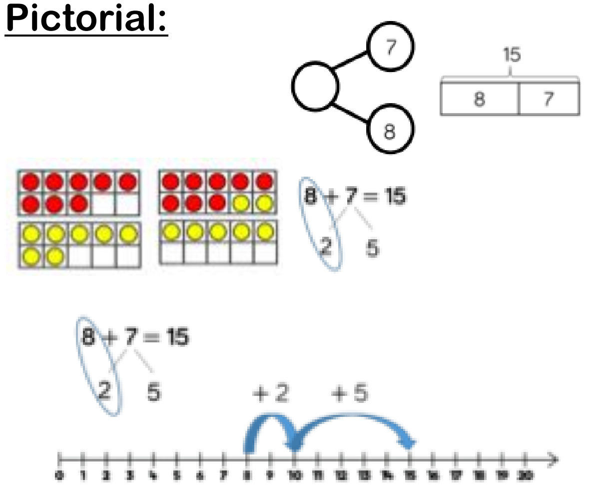
Concrete:



When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. Different manipulatives can be used to represent this exchange. Use concrete representations alongside number lines to support children in understanding how to partition their jumps.

The children should understand the importance of reaching a ten first before adding the rest. This will come from their understanding of their number bonds to 10.

Pictorial:



When moving to pictorial, the children should see the pictorial representation alongside the concrete at first before exploring new calculations with the pictorial representations alone. Number lines and tens frames are a key resource at this stage. If the children are drawing tens frames, the counters should be drawn as different representations, such as, crosses and circles. Use empty number lines and get the children to start the number line from the largest number and then count on from there – not from 0.

Abstract:

$$\boxed{8 + 7 = 15}$$

$$7 + \square = 15$$

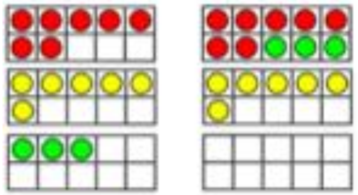
When the children are first introduced to the abstract representation, it should be alongside the pictorial representations. Once they are secure, they can solve new abstract calculations.

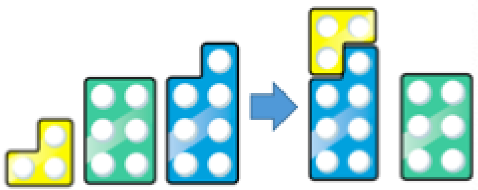
Stage 3

Add three 1-digit numbers

Example questions: 7 + 6 + 3 = 16

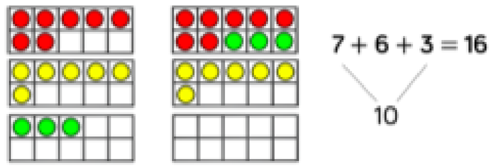
Concrete:

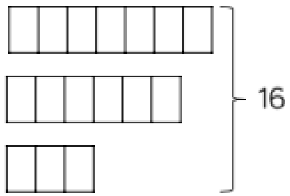


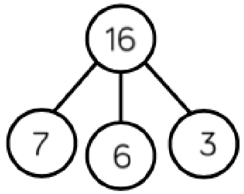


When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add numbers more effectively. For children to truly embed this concept they need to see the pattern through concrete resources first. For example, seeing that the Numicon of 3 first together with the Numicon piece of 7 to form and Numicon 10 piece. Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers. This supports children in their understanding of commutativity.

Pictorial:







When the children are using pictorial representations, they should us them in conjunction with the concrete initially. If possible, use pictorial representations that mirror the concrete resources. For example, if they have used tens frames in the concrete, then they should be drawing (or looking at) tens frames during the pictorial. Then they should be moving on to new questions with different pictorial representations.

Abstract

$7 + 6 + 3 = 16$

10

$\boxed{7 + 6 + 3 = 16}$

$? + 6 + 3 = 16$

At this stage, as with the concrete and pictorial, the children should be looking for their number bonds to 10. If they cannot identify them then they need to go back to the concrete so that they can visually see the importance of number bonds.

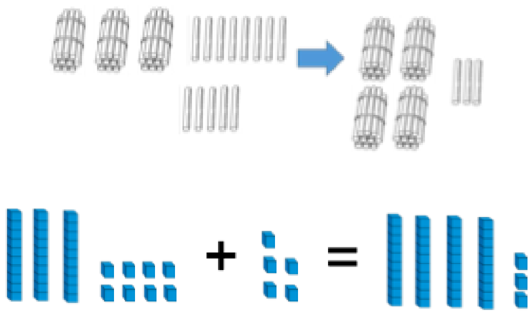
If the children are not using their number bonds, it is important to stop them and make it explicit for them. This is a key concept that should not be missed or overlooked.

Stage 4

Add 1-digit and 2-digit numbers to 100

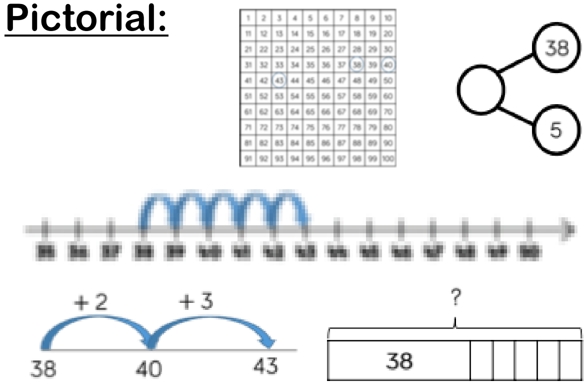
Example question: 38 + 5 = 43

Concrete:



When adding single digits to a two-digit number, children should be encouraged to count on from the larger number to the next ten (when appropriate). When using concrete resources, it should be made clear to the children that they can make another lot of ten and then add the rest of the ones. As shown above (38 + 2 = 40 and then 40 + 3 = 43). For children to deeply understand this concept, they need to see and do it physically first. Straws or lolly sticks can support children to find the number bonds to 10.

Pictorial:



Once this concept is understood using concrete resources, the children should use pictorial representations alongside the concrete. They should apply their knowledge of number bonds to add more effectively. For example, 8 + 5 = 13 so 38 + 5 = 48. Children should be continuously reminded about using their number bonds. If they do not have a secure understanding of how to use them, they should continue using concrete resources to support their learning and a number bonds to 10 intervention should be in place. When drawing base 10, children should use lines (tens) and crosses (ones).

Abstract:

$$\boxed{38 + 5 = 43}$$

$$38 + ? = 43$$

$$43 = ? + 5$$

Once the children have used and understood the concrete and pictorial, they should see the abstract alongside them. To have a deep understanding of this concept, the children should be able to solve missing number problems and understand that the answer and equal sign can be on either side of the calculation.

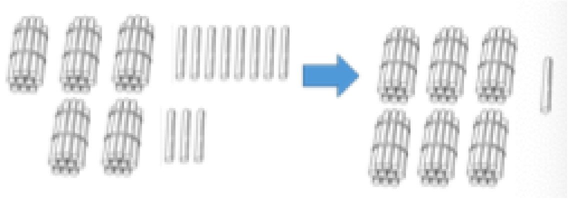
Partitioning the 2-digit number into tens and ones before adding the 1-digit number. Again, refer to number bonds. For example, 38 becomes 30 and 8; 8 + 5 = 13; 30 + 13 = 43.

Stage 5

Add two 2-digit numbers to 100

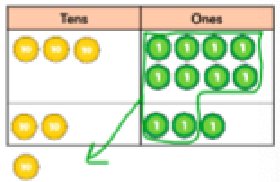
Example questions: 38 + 23 = 61

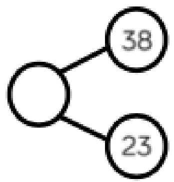
Concrete:

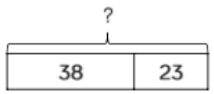


At this stage, the concrete, pictorial and abstract should be being using alongside one another even more. Encourage children to use the formal column method when calculating alongside straws, lolly sticks, base 10 or place value counters. As numbers become larger, straws and lolly sticks become less effective.

Pictorial:







As with the concrete, when the children are using pictorial representations, they should also have the abstract representation alongside it. Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to complete the calculation more effectively.

Abstract:

$$\boxed{38 + 23 = 61}$$

$$\begin{array}{r} 38 \\ +\,23 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$$

$$38 + ? = 61$$

$$61 = ? + 23$$

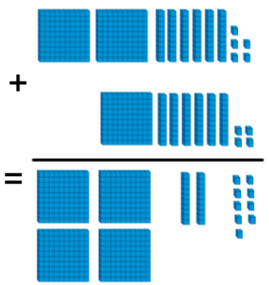
As with the concrete and pictorial, the children should be looking for number bonds to find the next multiple to 10 to make calculations easier. When writing the formal written method, it should be made explicit that the children start with the ones column and then move left to the tens column. The children should place the exchanged digit at the bottom of the column method. To ensure a deep understanding, the children should be confident and competent completing missing number problems and understand that the equals sign can go on either side of the calculation.

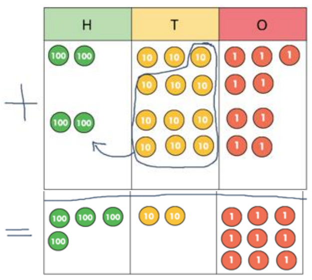
Stage 6

Add numbers with up to 3 digits

Example question: 265 + 164 = 429

Concrete:

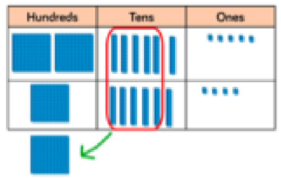


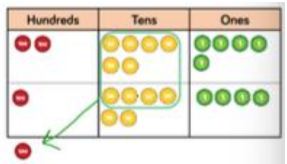


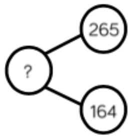
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure children write out their calculations alongside any concrete resources so they can see the links to the written column method, or that the abstract is visible to them somewhere. Plain counters on a place value grid can also be used to support learning.

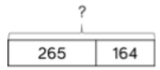
This is where it needs to be made explicit to the children that, as when there are 10 ones, we exchange 1 ten, when there are 10 tens, we exchange 1 hundred.

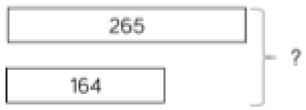
Pictorial:











If the children have used base 10 resources as their concrete manipulatives, then they should use base 10 as their pictorial. Base 10 and place value counters are key to the children understanding exchanging. They should be using the concrete alongside the pictorial – or the digital tools on the White Rose website.

Abstract:

265 + 164 = 429

```
  265
+ 164
  429
   1
```

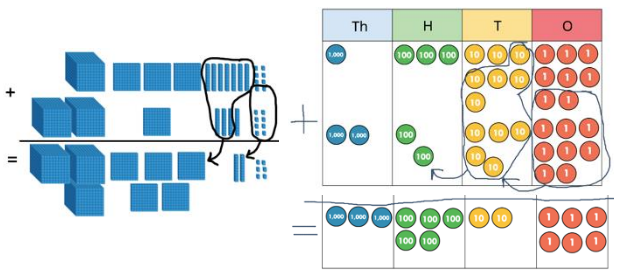
Ensure children write out their calculations alongside any concrete and pictorial resources so they can see the links to the written column method. This supports their understanding of exchanging when they are completing abstract calculations. When exchanging, the children need to put the exchanged digit underneath the formal column written method. To ensure a deep understanding, the children should be confident and competent doing missing number problems and understand that the equals sign can go on either side of the calculation.

Stage 7

Add numbers with up to 4 digits

Example questions: 1,378 + 2,148 = 3,526

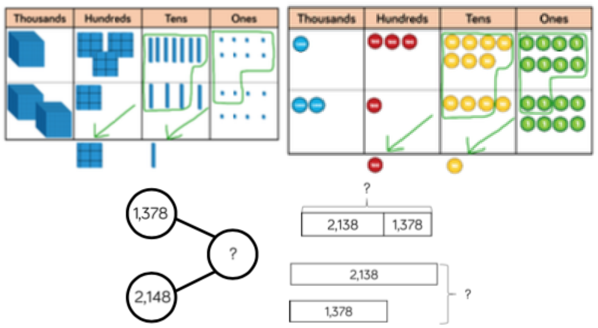
Concrete:



Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits. Ensure children write out their calculations alongside any concrete resources, so they can see the links to the written method, or that the abstract is visible to them somewhere. Plain counters on a place value grid can also be used to support learning.

This is where it needs to be made explicit to the children that, as when there are 10 ones, we exchange 1 ten, when there are 10 tens, we exchange 1 hundred and so on…

Pictorial:



If the children have used base 10 resources as their concrete manipulatives, then they should use base 10 as their pictorial. Base 10 and place value counters are key to the children understanding exchanging. They should be using the concrete alongside the pictorial – or the digital tools on the White Rose website.

Abstract:

	1	3	7	8
+	2	1	4	8
	3	5	2	6
		1	1	

$$1{,}378 + 2{,}148 = 3{,}526$$

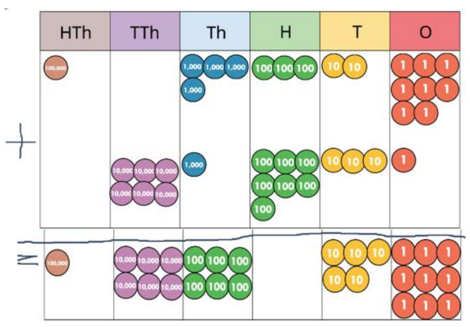
Ensure children write out their calculations alongside any concrete and pictorial resources so they can see the links to the written column method. This supports their understanding of exchanging when they are completing abstract calculations. When exchanging, the children need to put the exchanged digit underneath the formal column written method. To ensure a deep understanding, the children should be confident and competent doing missing number problems and understand that the equals sign can go on either side of the calculation.

Stage 8

Add numbers with more than 4 digits

Example question: 104,328 + 61,731 = 166,059

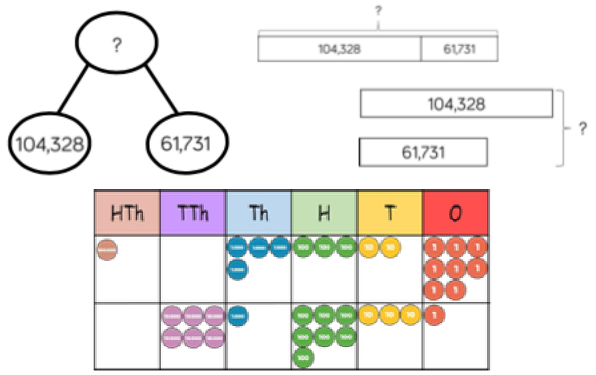
Concrete:



Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits. At this stage, place value counters can be used to both support the learning as well as extend it. Higher ability children often find it had to demonstrate mathematical concepts accurately when using concrete resources.

Again, it needs to be made explicit to the children that, as when there is 10 ones, we exchange 1 ten, when there is 10 tens, we exchange 1 hundred and so on…

Pictorial:



Using bar models and part-whole models can help children to understand that calculations work in a number of ways, e.g. the inverse. They can also support the children's understanding of the value of numbers - bigger numbers will have a bigger section of the bar model.

Drawing out place value counters are a good step for demonstrating that the children truly understand how exchanging works.

Abstract:

104,328 + 61,731 = 166,059

1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
		1			

104,328 + = 166,059

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers effectively.

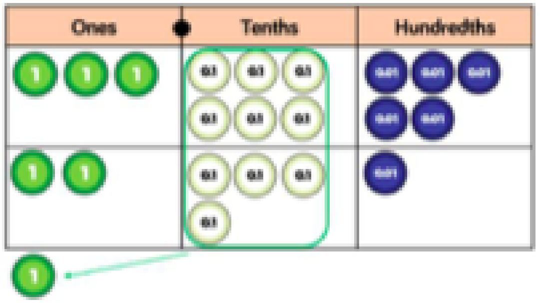
When exchanging, the children need to put the exchanged digit underneath the formal column written method. To ensure a deep understanding, the children should be confident and competent doing missing number problems and understand that the equals sign can go on either side of the calculation. They should also understand the inverse.

Stage 9

Add with up to 3 decimal places

Example question: 3.65 + 2.41 = 6.06

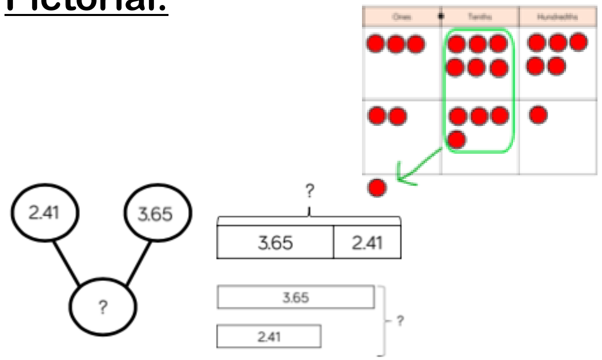
Concrete:



Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places. Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures. Money should be used as a concrete resource.

Base 10 can also be used to demonstrate decimals. The 1,000 cubes represent 1 whole; the 100 square becomes 10ths; the 10 stick becomes 100ths and the ones cubes become the 1,000ths.

Pictorial:



Drawing out place value counters support the children's learning as it is a familiar resource. It should be made clear to the children that the process of addition works in exactly the same way as addition with whole numbers. It needs to be made clear that the decimal should always be placed in the correct position before completing the calculation. Place value charts are a good step for demonstrating that the children truly understand how exchanging works in decimals.

Abstract:

$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline {\scriptstyle 1} \end{array}$$

$$\boxed{3.65 + 2.41 = 6.06}$$

When exchanging, the children need to put the exchanged digit underneath the formal column written method. To ensure a deep understanding, the children should be confident and competent doing missing number problems and understand that the equals sign can go on either side of the calculation. They should also understand the inverse.

Subtraction

EYFS:	Year 1:	Year 2:	Year 3:	Year 4:	Year 5:	Year 6:
Develop a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers.	Read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs. Represent and use number bonds and related subtraction facts within 20. Subtract one-digit and two-digit numbers to 20, including zero. Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9.	Recall and use subtraction facts to 20 fluently and derive and use related facts up to 100. Subtract numbers using concrete, pictorial and abstract representations, including: -A two-digit number and ones. -A two-digit number and tens. -Two two-digit numbers. To recognise and understand the inverse.	Subtract numbers including: -A three-digit. number and ones -A three-digit. number and tens -A three-digit number and hundreds. Subtract numbers with up to three digits, using formal written methods of columnar subtraction.	Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate.	Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction).	Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction).

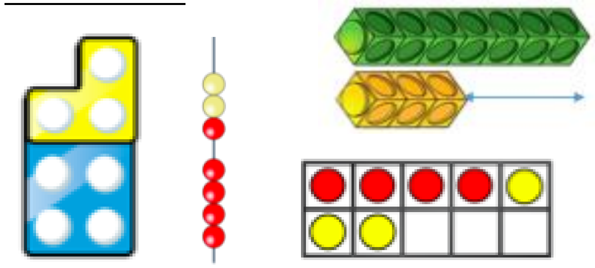
Key Vocabulary:	**STEM Sentences:**
Difference – The numerical difference between two numbers is found by comparing the quantity in each group. **Exchange** – Change a number or expression for another of an equal value. **Minuend** – A quantity or number from which another is subtracted. **Partitioning** – Splitting a number into its component parts. **Reduction** – Subtraction as take away. **Subitise** – Instantly, recognising the number of objects in a small group without needing to count. **Subtrahend** – A number to be subtracted from another.	'One less than ____ is ____.' 'I know that ____ subtract ____ is equal to ____ ; therefore, ____ subtract ____ is equal to ____.' '____ subtract ____ is equal to ____.' 'We line up the ones; ____ ones subtract ____ ones equals ____ ones. We line up the tens; ____ tens subtract ____ tens equals ____ tens.' Etc. 'We know there are ten hundreds in one thousand, so ____ hundred subtract ____ hundred is equal to ____ thousand ____ hundred.'

Stage 1

Subtract 1-digit numbers within 10

Example question: 7 – 3 = 4

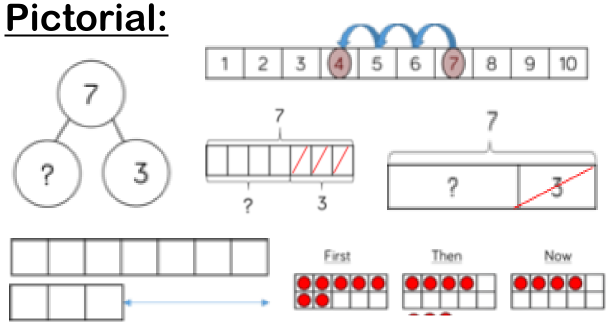
Concrete:



Cubes and tens frames are excellent concrete resources to demonstrate reduction. Cubes or counters can be physically taken away from the amount.

Ten frames, cubes, and bead strings support reduction.

Pictorial:



Bar models should be used in conjunction with the concrete representations. The pictorial bar model is almost exactly the same and the cubes (concrete).

Number lines can support children's understanding of counting backwards - reduction.

Part-whole models, bar models, ten frames and number shapes support partitioning.

Abstract:

$7 - 3 = 4$

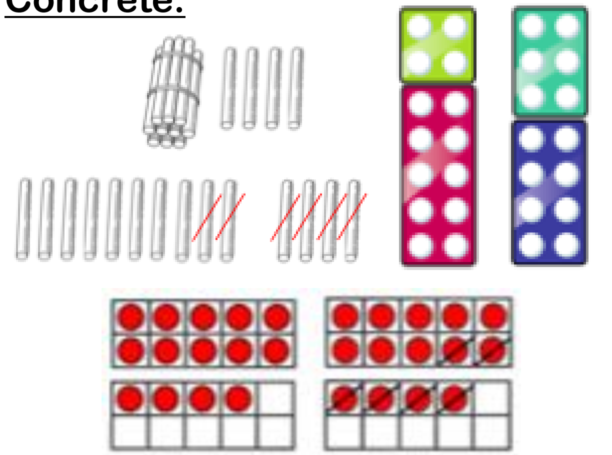
The abstract calculation should be shown alongside the concrete resource so that the children can clearly see the relationship been the abstract and the concrete.

Stage 2

Subtract 1 and 2-digit numbers within 20

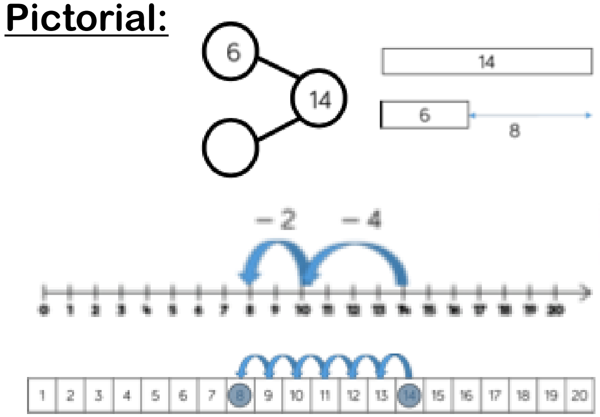
Example questions: 14 – 6 = 8

Concrete:



When subtracting 1-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Tens frames, cubes, base 10 and number lines are particularly useful for this.

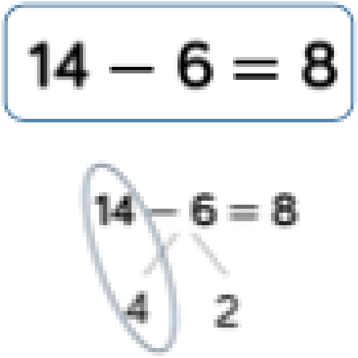
Pictorial:



Counting back on number lines help support the children's understanding of subtraction – reduction. Again, children should be looking for number bonds again. They should be looking for the amount to subtract to reach the previous multiple of 10.

Bar models and part-whole models support the children's understanding of partitioning. This also helps them to see the relationship between addition and subtraction.

Abstract:



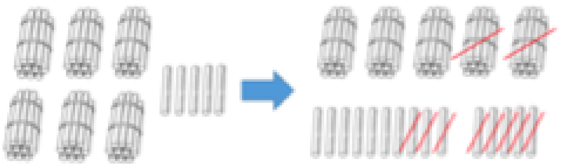
The abstract calculation should be shown alongside the concrete resource so that the children can clearly see the relationship been the abstract and the concrete.

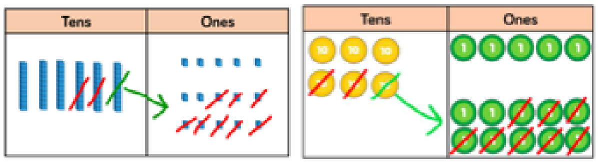
Stage 3

Subtract 1 and 2-digit numbers to 100

Example questions: 65 – 28 = 37

Concrete:

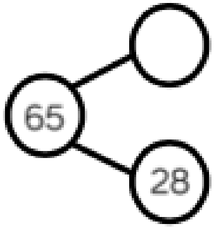


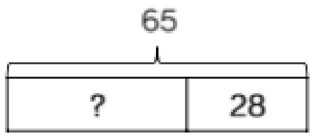


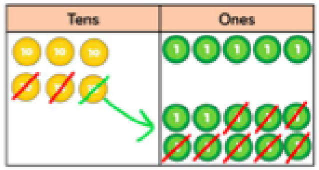
At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 and place value counters. As numbers become larger, straws become less effective.

Even when using concrete resources, it should be made explicit to the children that they should start by taking away the ones and then move to the left. This is where the children need to understand that 1 ten equals 10 ones. This exchanging must be modelled clearly to the children.

Pictorial:







Exchanging from another column is a hard concept for children to understand; therefore, they should use pictorial representations alongside the concrete at this point to secure their understanding. The abstract should also be visible.

Bar models and part-whole models support the children's understanding of partitioning. This also helps them to see the relationship between addition and subtraction.

Abstract:

$$\boxed{65 - 28 = 37}$$

$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}5 \\ -\ 28 \\ \hline 37 \\ \hline \end{array}$$

At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 and place value counters.

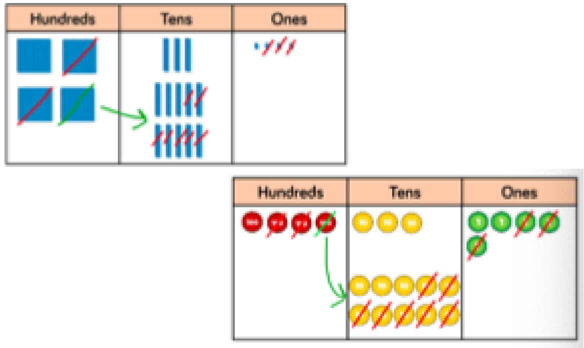
When completing the formal written method, the children should be placing the exchanged number at the top left of the original digit – as seen above. IT should be made clear that they need to start from the ones column and then move to the left though the remaining columns.

Stage 4

Subtract numbers with up to 3 digits

Example questions: 435 – 273 = 262

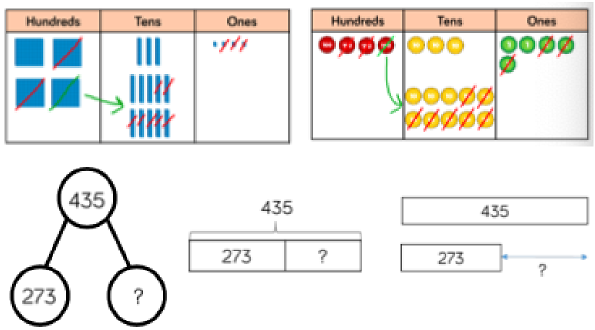
Concrete:



Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits. Plain counters on a place value grid can also be used to support learning.

Ensure children write out their calculations alongside any concrete resources so they can see the links to the formal written method.

Pictorial:



Alongside the concrete and abstract, the children should be drawing out the pictorial.

Bar models and part-whole models support the children's understanding of partitioning. This also helps them to see the relationship between addition and subtraction.

Abstract:

$$435 - 273 = 262$$

$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}35 \\ -\;273 \\ \hline 262 \\ \hline \end{array}$$

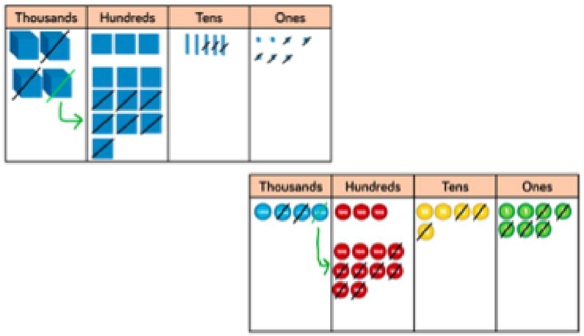
When completing the formal written method, the children should be placing the exchanged number at the top left of the original digit – as seen above. The children should be reminded that the start from the ones column and then move to the left though the remaining columns.

Stage 5

Subtract numbers with up to 4 digits

Example questions: 4,357 – 2,735 = 1,622

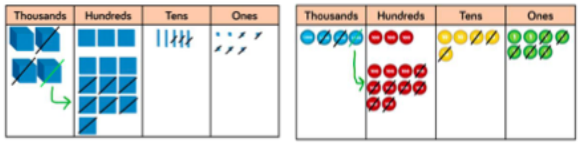
Concrete:

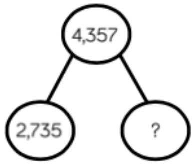


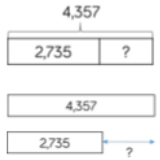
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 4 digits. Plain counters on a place value grid can also be used to support learning.

Ensure children write out their calculations alongside any concrete resources so they can see the links to the formal written method.

Pictorial:







Alongside the concrete and abstract, the children should be drawing out the pictorial.

Bar models and part-whole models support the children's understanding of partitioning. This also helps them to see the relationship between addition and subtraction.

Abstract:

$\boxed{4{,}357 - 2{,}735 = 1{,}622}$

$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}357 \\ -\ 2735 \\ \hline 1622 \\ \hline \end{array}$$

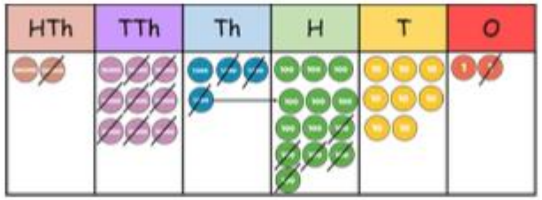
When completing the formal written method, the children should be placing the exchanged number at the top left of the original digit – as seen above. The children should be reminded to start from the ones column and then move to the left though the remaining columns.

Stage 6

Subtract numbers with more than 4 digits

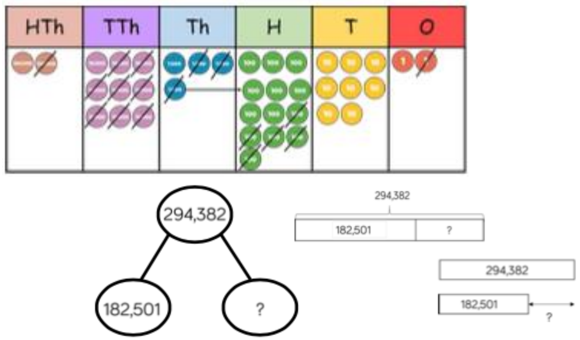
Example questions: 294,382 – 182,501 = 111,881

Concrete:



Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

Pictorial:



Bar models and part-whole models support the children's understanding of partitioning. This also helps them to see the relationship between addition and subtraction.

Abstract:

294,382 – 182,501 = 111,881

	2	9	$^{3}\cancel{4}$	$^{1}3$	8	2
–	1	8	2	5	0	1
	1	1	1	8	8	1

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

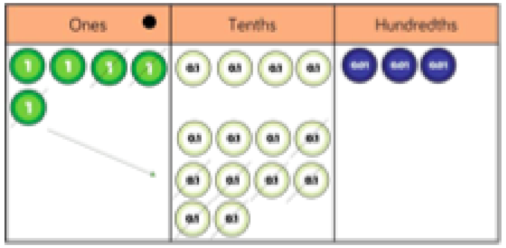
When completing the formal written method, the children should be placing the exchanged number at the top left of the original digit – as seen above. The children should be reminded that the start from the ones column and then move to the left though the remaining columns.

Stage 7

Subtract with up to 3 decimal places

Example questions: 5.43 – 2.7 = 2.73

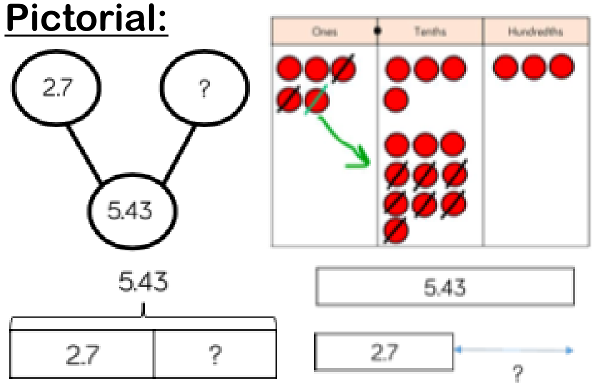
Concrete:



Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 or 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this in context when subtracting money and other measures that can be used as a concrete resource.

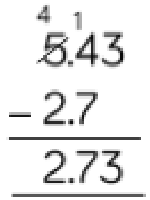
Pictorial:



Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this in context when subtracting money and other measures that can be used as a pictorial resource.

Abstract:

$$\boxed{5.43 - 2.7 = 2.73}$$



When completing the formal written method, the children should be placing the exchanged number at the top left of the original digit – as seen above. The children should be reminded that the start from the furthest column on the right (dependent on the number of decimal places) and then move to the left through the remaining columns.

It should be made clear to the children that they need to put the decimal point in their answer before they start working out – to avoid errors.

Multiplication						
EYFS:	**Year 1:**	**Year 2:**	**Year 3:**	**Year 4:**	**Year 5:**	**Year 6:**
Doubling, sharing and grouping.	Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=). Show that multiplication of two numbers can be done in any order (commutative).	Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.	Recall multiplication facts for tables up to 12 × 12. Multiply mentally, including multiplying by 0 and 1 and multiplying together three numbers. Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.	Identify and understand multiples and factors; square numbers and cube numbers; prime numbers and composite (nonprime) numbers. Multiply numbers up to 4 digits by a 1-digit and 2-digit numbers using the formal written method of short division, long division and interpret remainders. Multiply whole numbers by 10, 100 and 1000.	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. Identify common factors, common multiples and prime numbers.

Key Vocabulary:

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Number can be multiplied in any order.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor.

STEM Sentences:

'There are ____ apples; the total value is ____.'

'There are ____ coins. Each coin has a value of ____ p. This is ____ p.'

'There are ____ equal groups of ____. There are ____ ____ in each group. There are ____ groups of ____.'

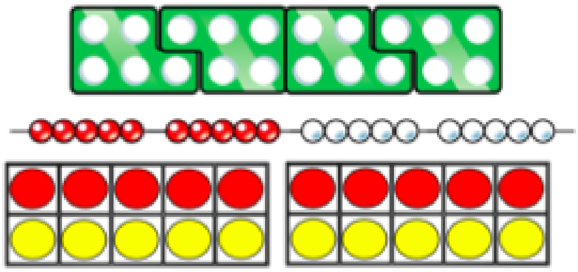
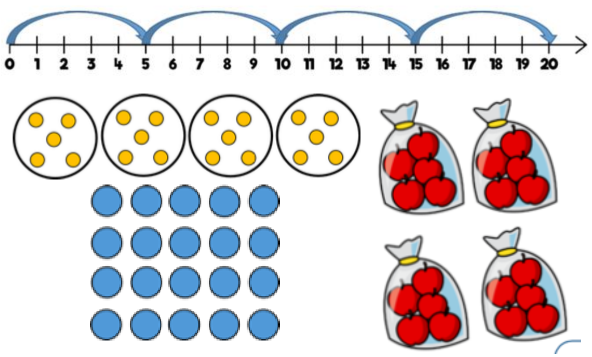
'The product of ____ and ____ is equal to the product of ____ and ____.'

'____ times ____ ones is equal to ____ ones, so ____ times ____ tenths is equal to ____ tenths.'

Stage 1

Solve 1-step problems using multiplication

Example question: 5 x 4 = 20

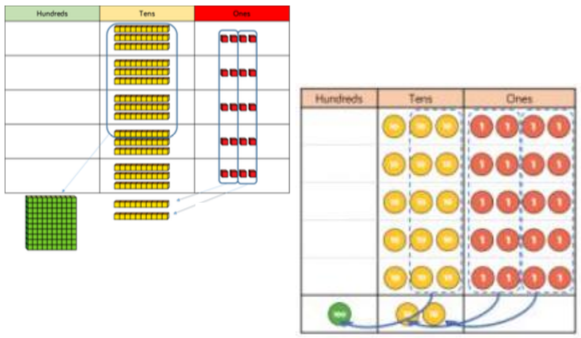
Concrete:	**Pictorial:**	**Abstract:**
		One bag holds 5 apples. How many apples do 4 bags hold? $5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$
Children should represent multiplication as repeated addition in many different ways before seeing the abstract multiplication sign (x) as they should already be familiar with the addition sign and so it reduces their cognitive load. They need to understand that multiplication is just repeated addition from the moment this concept is introduced to them. In Year 1, children use concrete representations to solve problems. They are not expected to record multiplication formally. The abstract of repeated addition should be visible or written out by the child at the same time as the concrete.	In Year 1, children use pictorial representations to solve problems. They are not expected to record multiplication formally. They should be exposed to a number of different representations of repeated addition before moving to the abstract multiplication sign (x). While looking at the pictorial, the abstract of repeated addition should be visible or written out by the child. When the children are introduced to the abstract multiplication sign, they should also have the repeated addition visible as well, with the pictorial.	When the children are starting to work with the abstract multiplication sign, they should also have the repeated addition visible as well. The children need to understand that the multiplication sign means how many of that number there is, displaying the link between repeated addition and multiplication.

Stage 2

Multiply 2-digit numbers by 1-digit numbers

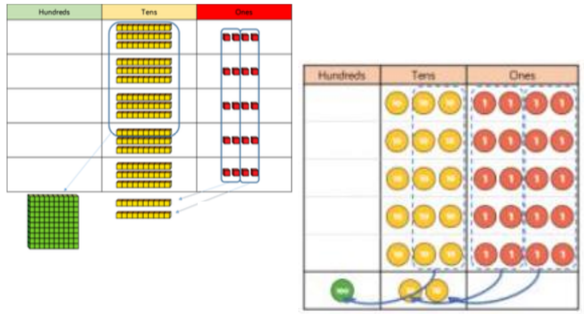
Example question: 34 x 5 = 170

Concrete:



The place value and base 10 resources should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge. This should support their knowledge that multiplication is repeated addition.

Pictorial:



The place value and base 10 resources should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge. This should support their knowledge that multiplication is repeated addition

Abstract:

$34 \times 5 = 170$

	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

	H	T	O
		3	4
×			5
	1	7	0
	1	2	

Teachers may decide to look at the expanded column method before moving on to the short multiplication method. Children should have a strong understanding of their times tables at this stage. The exchanged number should always be at the bottom (as will addition). If the children do not, the questions they are given should reflect their times table knowledge.

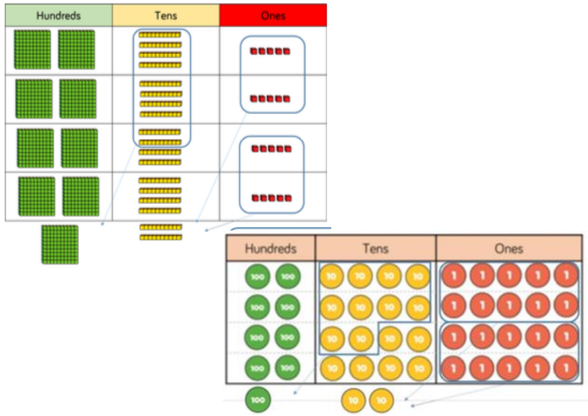
If the child only knows the 2, 5 and 10 times tables, the 1-digit number should only be 2, 5 or 10. They are learning the method not their times tables, reducing their cognitive load.

Stage 3

Multiply 3-digit numbers by 1-digit numbers

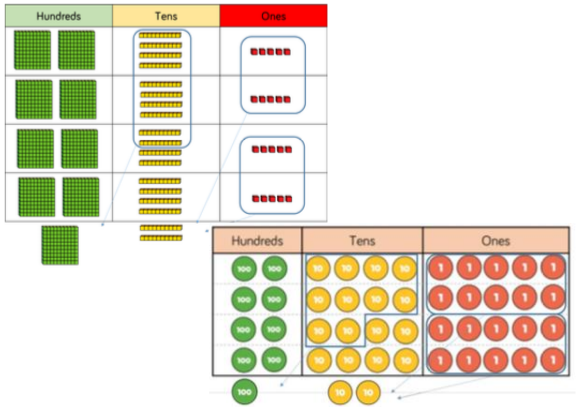
Example question: 245 x 4 = 980

Concrete:



When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal method. Base 10 and place value counters continue to support the understanding of the written method. As the numbers become larger, it becomes harder to demonstrate exchanges. At this point, questions need to be thought about carefully. Limit the number of exchanges needed in the question and move children away from resources when multiplying larger numbers.

Pictorial:



When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal method. Base 10 and place value counters continue to support the understanding of the written method. As the numbers become larger, it becomes harder to demonstrate exchanges. At this point, questions need to be thought about carefully. Limit the number of exchanges needed in the question and move children away from resources when multiplying larger numbers.

Abstract:

$245 \times 4 = 980$

	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

When using the formal written method, the children need to ensure that they put the exchanged number at the bottom (as with addition).

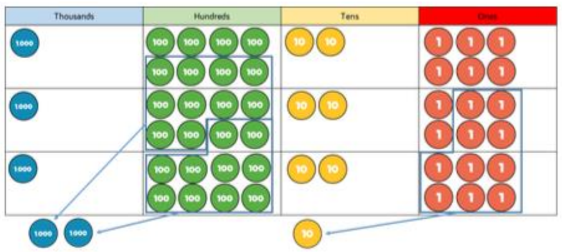
The children should be encouraged to use the formal method at this point, not the expanded method. However, the expanded method may be used to support children that are struggling with the concept.

Stage 4

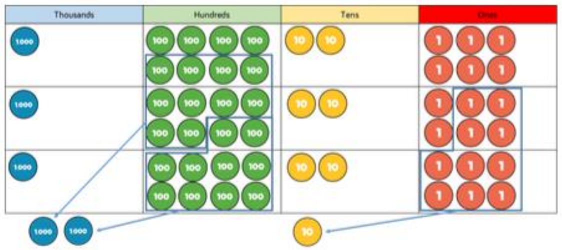
Multiply 4-digit numbers by 1-digit numbers

Example question: 1,826 x 3 = 5,478

Concrete:



Pictorial:



When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.

If the children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so the children can focus on the use of the written method.

Abstract:

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

$1{,}826 \times 3 = 5{,}478$

If the children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so the children can focus on the use of the written method.

When using the formal written method, the children need to ensure that they put the exchanged number at the bottom (as with addition).

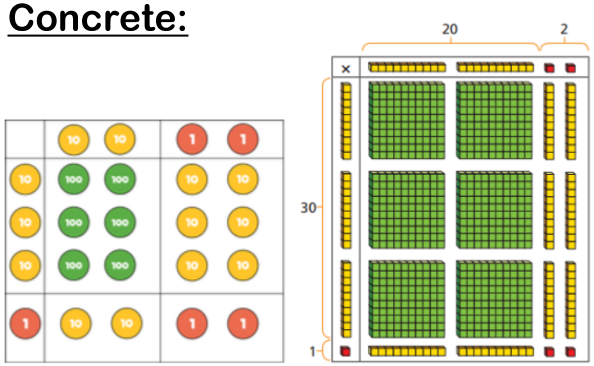
The children should not be using the expanded method at this point, unless absolutely necessary.

Stage 5

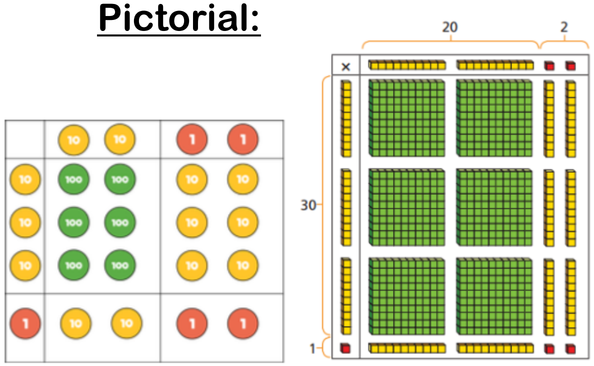
Multiply 2-digit numbers by 2-digit numbers

Example question: 22 x 31 = 682

Concrete:



Pictorial:



Abstract:

×	20	2
30	600	60
1	20	2

$$22 \times 31 = 682$$

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

When multiplying a multi-digit number by a 2-digit number, use the area model (using base 10) to help the children understand the size of the numbers thy are using. This links to finding the area of a rectangle by finding the space covered by the Base 10. This is an unusual method and teachers should ensure they understand and are comfortable using this method. Once the children are comfortable using the Base 10, they should then apply this knowledge to the place value counters. Place value counters are also easier for the children to draw into their books.

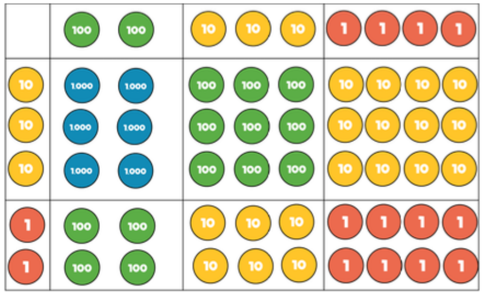
The grid method matches the area model as an initial written method before moving on to the formal written multiplication method. When the children are first introduced to the abstract grid method, it should be alongside the area model using Base 10.

Stage 6

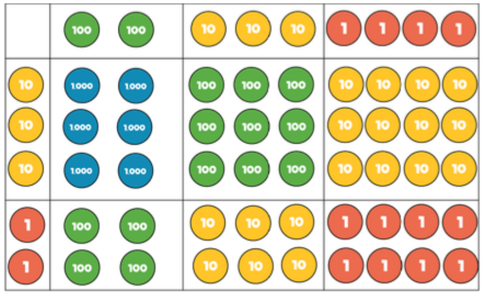
Multiply 3-digit numbers by 2-digit numbers

Example question: 234 x 32 = 7,488

Concrete:



Pictorial:



The children can continue to use the area model when multiplying 3-digit by 2-digis. Place value counters become more effective to use but Base 10 can be used to highlight the size of numbers, as with the previous stage.

Abstract:

$$234 \times 32 = 7{,}488$$

×	200	30	4
30	6,000	900	120
2	400	60	8

Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$_{1}7$	$_{1}0$	2	0
7	4	8	8

The children should be encouraged to move towards the formal written method, seeing the links with the grid method.

Any exchanges should be placed in the box to the left; the next place value column – as seen above.

Stage 7

Multiply 4-digit numbers by 2-digit numbers

Example question: 2,739 x 28 = 76,692

Concrete:

Pictorial:

Abstract:

$$2{,}739 \times 28 = 76{,}692$$

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 (2)	1 (5)	9 (3)	1 (7)	2
5 (1)	4	7 (1)	8	0
7	6	6	9	2

When multiplying 4-digitd by 2-digits, children should be confident in the written method. If they are still struggling, provide multiplication grids to support when they are focusing on the use of the method. Consider where exchanged digits are placed and make this consistent.

Division

EYFS:	Year 1:	Year 2:	Year 3:	Year 4:	Year 5:	Year 6:
Doubling, sharing and grouping.	Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs. Show that division is not commutative. Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts.	Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. Solve problems, including missing number problems, involving division.	Recall division facts for multiplication tables up to 12 × 12. Divide mentally, including dividing by 1. Recognise and use factor pairs and commutativity. Solve problems involving multiplying two-digit numbers by one digit, integer.	Identify and understand multiples and factors; square numbers and cube numbers; prime numbers and composite numbers. Divide numbers up to 4 digits by a one- or two-digit number using a formal written method (short division), including long division for two-digit numbers – including remainders. Divide whole numbers by 10, 100 and 1000.	Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Key Vocabulary:

Array – An ordered collection of counters, cubes or other item in rows and columns.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make product.

Partitioning – Splitting a number into its component parts.

Quotient – The result of a division.

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor.

STEM Sentences:

'There are ____ apples have been shared into ____ groups. There are ____ apples in each group.'

'There are ____ groups of ____; there are ____ altogether.'

'____ is divided into groups of ____. There are ____ groups.'

'____ is divided into ____ groups of ____.'

'____ is divided into ____ groups of ____ with a remainder of ____.'

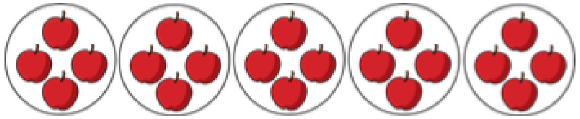
'____ divided by ten is equal to ____.'

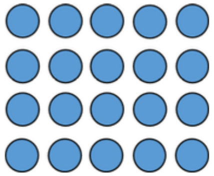
Stage 1

Solve 1-step problems using multiplication (sharing)

Example question: 20 / 5 = 4

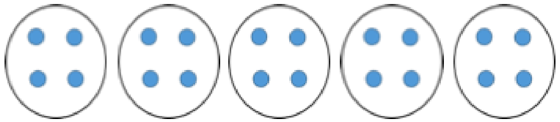
Concrete:

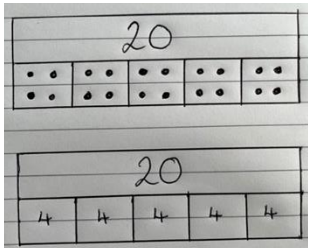




Children should solve problems by sharing concrete objects into equal groups. Sharing using concrete resources should be secure before moving to pictorial representations. In Year 1, children are not expected to record division formally; they are only expected to use concrete and pictorial representations. The children should not be introduced to the word division; they should only be using the word '**sharing'**. In Year 2, children are introduced to the division symbol. The abstract should be shown alongside the concrete and/or pictorial representations.

Pictorial:





Pictorial representations should only be used once the children understand the concept of sharing using concrete resources. A good transition from the pictorial understanding to the abstract is using bar models. They can start by dividing dots into the lower box on a bar model; then they can do the same activity using abstract numbers, as seen above.

Abstract:

$$20 \div 5 = 4$$

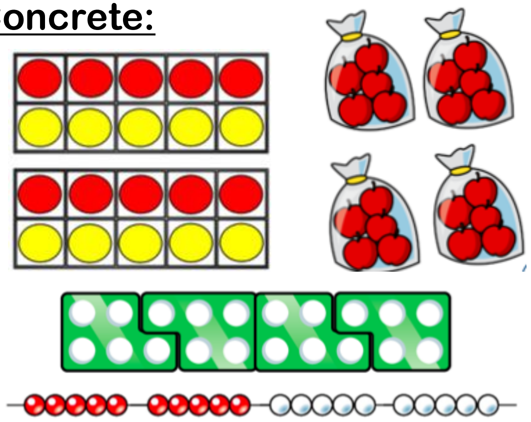
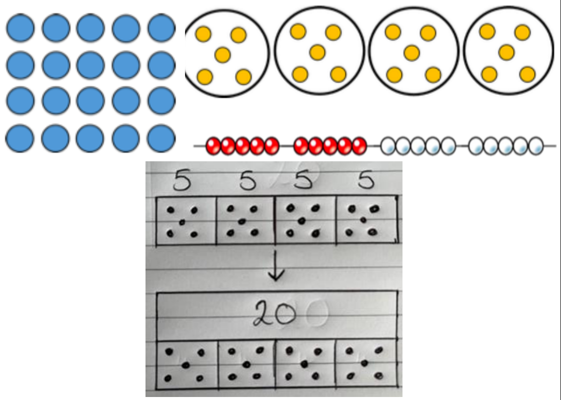
There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?

The children should be introduced to the division symbol in Year 2. When this is first introduced, the abstract should be shown alongside the concrete and pictorial. The children need to see the relationship between the abstract symbol and the process of sharing.

Stage 2

Solve 1-step problems using division (grouping)

Example question: 20 / 5 = 4

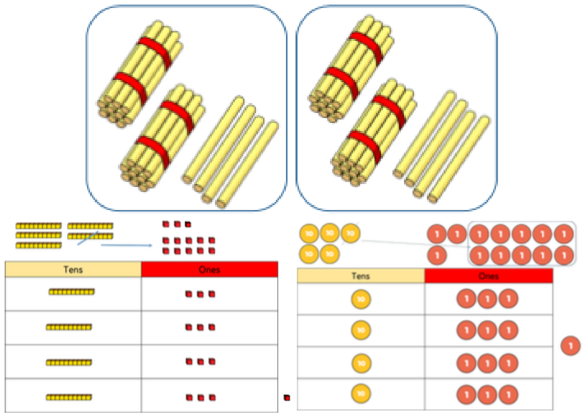
Concrete:	**Pictorial:**	**Abstract:**
		$20 \div 5 = 4$ There are 20 apples altogether. They are put in bags of 5. How many bags are there?
Children should solve problems by grouping the objects and then counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They use concrete representations in fixed groups, such as, Numicon which helps to show the link between multiplication and division.	The pictorial should be use alongside the concrete. Initially, the children should be drawing the concrete resource they have been using. Bar models can also be used to support grouping as well – as seen above.	The children should be introduced to the division symbol in Year 2. When this is first introduced, the abstract should be shown alongside the concrete and pictorial. The children need to see the relationship between the abstract symbol and the process of grouping.

Stage 3

Divide 2-digit by 1-digit (sharing with exchange and remainders)

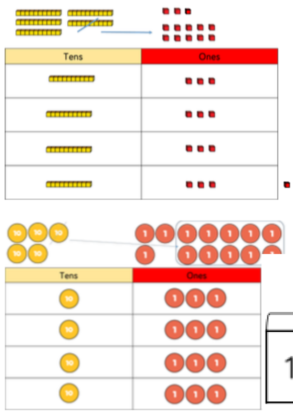
Example question: 53 / 4 = 13r1

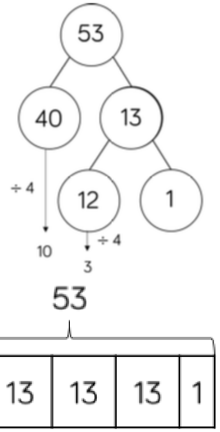
Concrete:



When dividing larger numbers, including those that involve exchanging and remainders, children should use concrete manipulatives that allow them to partition into tens and ones. Straws, Base 10 and place value counters can all be used to share numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows. This will also highlight remainders, as they will be left outside the grid once the equal groups have been made.

Pictorial:





Part-whole models can provide children with clear written method that matches the concrete. Flexible partitioning in a part-whole model supports children when they are using concrete resources outside the place value grid before sharing the tens and ones equally between the rows.

Abstract:

$$53 \div 4 = 13\text{ r}1$$

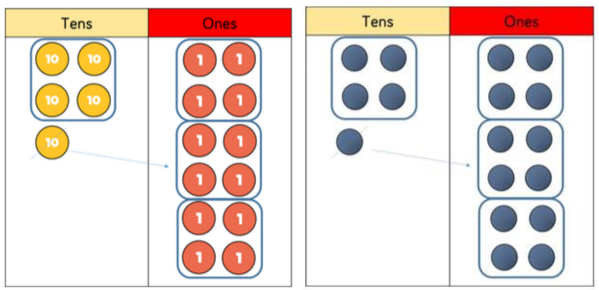
The abstract should always be shown with the concrete and pictorial representations. The children should already understand the division symbol. If they don't, they should be moved back a stage (this could be in the form of an intervention).

Stage 4

Divide 2-digit by 1-digit (grouping)

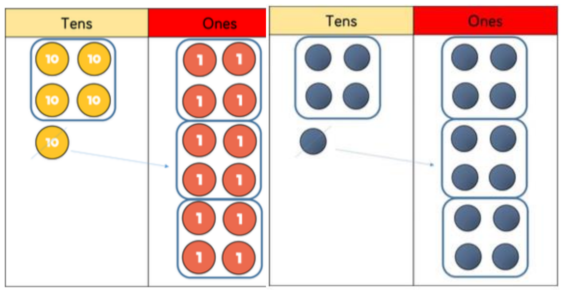
Example question: 52 / 4 = 13

Concrete:



When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor. Language is important here. Children should consider, **‘How many groups of 4 tens can we make?’** and, **‘How many groups of 4 ones can we make?’** Remainders can also be seen as they are left ungrouped. While the children are learning about the concrete, they should also have the abstract visible. The abstract should not be explained but it should be visible.

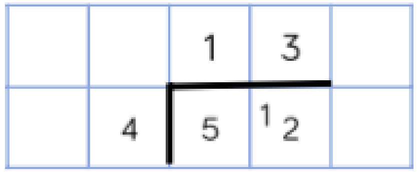
Pictorial:



When moving to the pictorial, the children should use representations that match the concrete resources they have been using. They should be drawing rings around the groups of counters. While the children are learning about the pictorial, they should also have the abstract visible. They abstract should not be explained but it should be visible.

Abstract:

$$\boxed{52 \div 4 = 13}$$



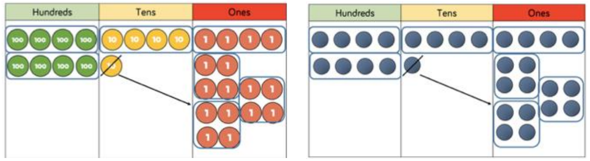
When the children are first introduced to the abstract representation, they should have the concrete and pictorial resources visible as well. They need to see the link between what they have been doing with the concrete/pictorial and how that is represented through the abstract.

Stage 5

Divide 3-digit by 1-digit (grouping)

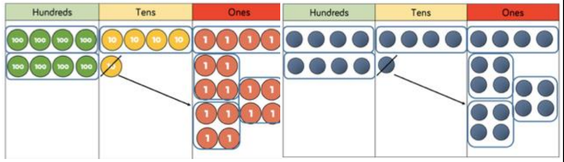
Example question: 856 / 4 = 214

Concrete:



Children should continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number. Place value counters or plain counters can be used on the place value grid to support this understanding.

Pictorial:



When pictorial representations are first used, they should be used in conjunction with the concrete resources. This should ensure that the understanding from using the concrete resources transfers to the pictorial representations. When moving to the pictorial, the children should be able to draw out their place value grid and fill in the counters, ensuring the draw around the groups.

Abstract:

$856 \div 4 = 214$

		2	1	4
	4	8	5	^{1}6

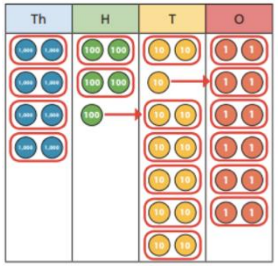
The abstract should be shown alongside the concrete and pictorial. This is to ensure that they understand what is happening when they are using the abstract.

Stage 6

Divide 4-digit by 1-digit (grouping)

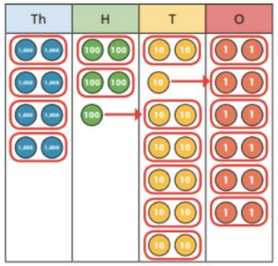
Example question: 8,532 / 2 = 4,266

Concrete:



Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit numbers. Children can also draw their own counters and group them through a more pictorial method. Children should be encouraged to move away from the concrete and pictorial representations when dividing numbers with multiple exchanges.

Pictorial:



The children should be able to represent what they have done in pictorial form. The easiest way to do this is to use place value counters for both.

Abstract:

$$8{,}532 \div 2 = 4{,}266$$

	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

The children should be using the abstract more when more exchanges occur. When they children are being taught this process, the teacher should model to them how to check their work for mistakes.

At this stage, when using the formal written method, short division, the children need to ensure they place the exchanged number in the top left-hand side of the box to the right.

Stage 7

Divide any number by 2-digit numbers (short division)

Example question: 432 / 12 = 36

Concrete:

Pictorial:

Abstract:

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

When children begin to divide up to 4-digits by 2 digits, the written methods become the most accurate as concrete and pictorial representations become less effective. Children can write our multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient (a result obtained by dividing one quantity by another) can be rounded as appropriate.

At this stage, when using the formal written method, short division, the children need to ensure they place the exchanged number in the top left-hand side of the box to the right.

Stage 8

Divide any number by 2-digit numbers (long division)

Example question: 432 / 12 = 36

Concrete:

Pictorial:

Abstract:

		0	3	6	
1	2	4	3	2	(×30)
	−	3	6	0	
			7	2	(×6)
	−		7	2	
				0	

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$432 \div 12 = 36$

$7{,}335 \div 15 = 489$

	0	4	8	9	
15	7	3	3	5	
−	6	0	0	0	(×400
	1	3	3	5	
−	1	2	0	0	(×80)
		1	3	5	
−		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Children are also taught to divide by 2-digit numbers using long division (depending on the value of the 2-digit number). They should write out multiples to support their calculations with larger remainders. They will also solve problems with remainders where the quotient can be rounded as appropriate. Children should set out their work, as seen above, unless the specific needs of the child warrant them completing long division questions in an alternative way.

Stage 9

Divide any number by 2-digit numbers (long division with remainders)

Example question: 372 / 15 = 24r12

Concrete:

Pictorial:

Abstract:

$$372 \div 15 = 24 \text{ r}12$$

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$

$2 \times 15 = 30$

$3 \times 15 = 45$

$4 \times 15 = 60$

$5 \times 15 = 75$

$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	–	3	0	0	
			7	2	
	–		6	0	
			1	2	

Complete the number track with the multiples of 15

15									

$$372 \div 15 = 24\,\frac{4}{5}$$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question. They can also answer questions where the quotient needs to be rounded according to the context. The children should also write out the number track for the multiple that they are dividing by. They can write it out however they like but it should be there to avoid mistakes.

Fractions

EYFS:	Year 1:	Year 2:	Year 3:	Year 4:	Year 5:	Year 6:
	Recognise, find and name a half as one of two equal parts of an object, shape or quantity. Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.	Recognise, find, name and write fractions 1/3 , 1/4 , 2/4 and 3/4 of a length, shape, set of objects or quantity. Write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and 1/2.	Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10. Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators. Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators. Recognise and show, using diagrams, equivalent fractions with small denominators. Add and subtract fractions with the same denominator within one whole [for example, 5/7 + 1/7 = 6/7]. Compare and order unit fractions, and fractions with the same denominators.	Recognise and show, using diagrams, families of common equivalent fractions. Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten. Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number. Add and subtract fractions with the same denominator Solve simple measure and money problems involving fractions and decimals to two decimal places.	Compare and order fractions whose denominators are all multiples of the same number. Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, 2/5 + 4/5 = 6/5 = 1 1/5]. Add and subtract fractions with the same denominator and denominators that are multiples of the same number. Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.	Use common factors to simplify fractions; use common multiples to express fractions in the same denomination. Compare and order fractions, including fractions > 1. Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions. Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8]. Divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6].

Key Vocabulary:

Numerator – the top digit of a fraction.

Denominator – the bottom digit of a fraction.

Proper Fractions – fractions that are less than 1.

Improper Fractions – fractions that are more than 1.

Mixed Number – a mixed number is a combination of a whole number and a proper fraction that is less than 1.

Equivalent – the same value.

Simplifying – dividing the digits in the fractions to their "smallest" form by using a common factor.

Compare – ordering or looking at the different values of fractions.

Integer – a whole number.

Unit Fractions – fractions where the numerator is 1.

Non-unit Fractions – fractions where the numerator is greater than 1.

STEM Sentences:

'The whole has been divided into ______ equal parts and we have ______ parts. This represents $\frac{?}{?}$.'

'I have folded my whole length of paper into ______ equal/unequal parts.'

'The parts are equal/unequal. I know this because the number of _____ in each part is the same.'

'If _____ is the whole, then _____ is part of the whole.'

'The denominator is ____ because the whole is divided into ____ equal parts.'

'There are _____ parts between zero and one. This means we are counting in units of ____.'

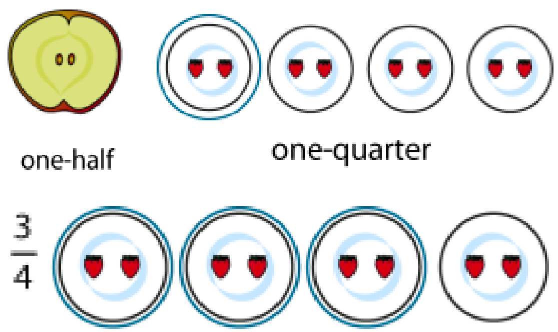
'Each interval on the line is divided into ____ equal parts. This allows us to count in ____.'

Stage 1

Recognise and understand the value of simple fractions

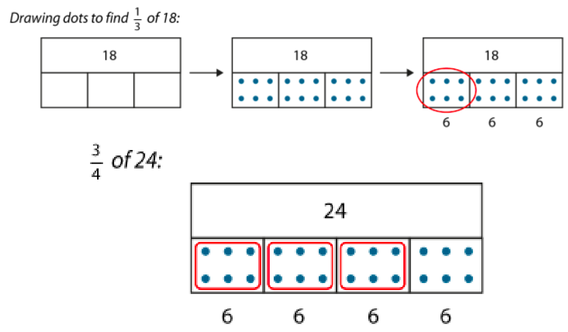
Example: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$

Concrete:



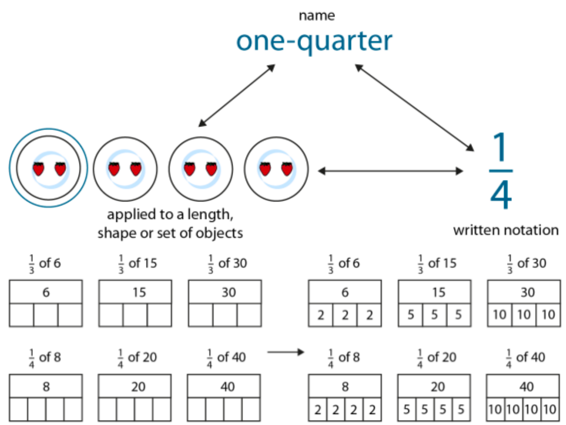
The language that should be used from the start should be **half, quarter and third**. Starting with concrete objects teach children to identify whether something has been split into $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$. They will then need opportunities to demonstrate that they can do this as well, e.g. "The 6 cubes are my whole, so these 3 cubes are my half".

Pictorial:



When looking at the pictorial, the children should still be able to identify whether something has been split into $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}$. They will then need to demonstrate this by drawing pictorial representations. They may draw things like shapes and then split them into equal parts.

Abstract:



When introducing the children to the abstract, take each fraction in turn and begin by naming it - e.g. one-half. Then describe it, for example, 'We know this is one-half because the apple has been divided into two equal parts and we have one of them.' Then show how to link this to the written notation – as seen in the diagram above.

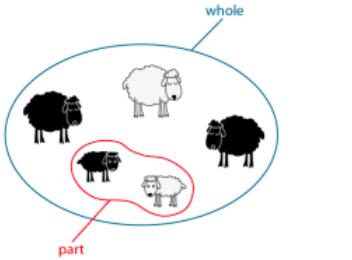
Stage 2

The part-whole relationship

Example: $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$

Concrete:

'If Europe is the whole, then the UK is part of the whole.'

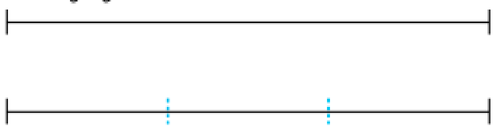


'If the group of sheep is the whole, then the lambs are part of the whole.'
'The part is smaller than the whole.'

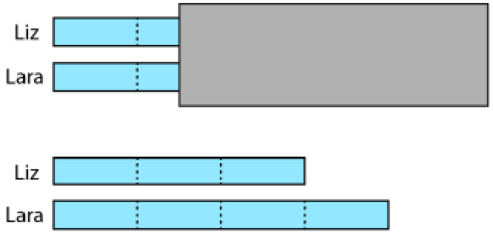
At this stage, the children do not need to use the term fraction immediately. The understanding here should be about something being part of a whole. Start big (Europe) and then 'zoom in' (e.g. UK, streets, animals then counters). If the children have a deep understanding of this concept, then they will have a better understanding of fractions as a whole later. This stage comes before the children move on to other fractions. It is important to use linear models at this early stage because, at later stages, children will need to understand fractions as numbers that can be positioned on a number line. Children can also cut out shapes and cut them into equal/unequal parts.

Pictorial:

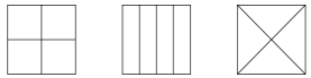
Dividing a given line into three:



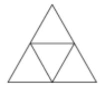
'Liz has folded her paper strip into three equal parts. Lara has folded her paper strip into four equal parts. Part of their strips are hidden. Whose paper strip is longer?'



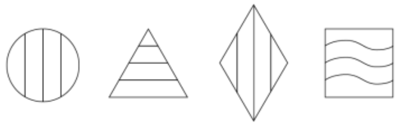
Squares divided into four equal parts in different ways:



An equilateral triangle divided into equal parts:



Shapes divided into parts of equal width but where the parts are not of equal size:



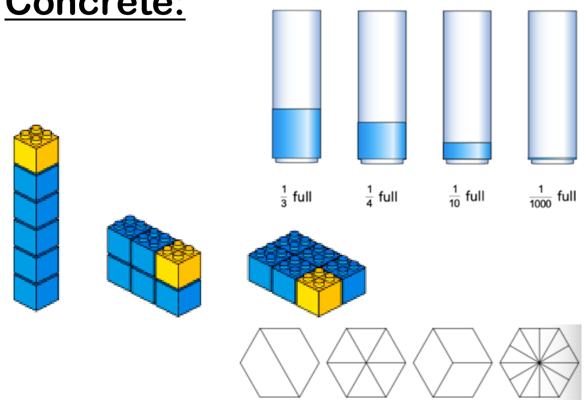
Most of the concrete resources can also be represented as pictorial representations. Using pictorial representations that match the concrete resources will help the children transfer their understanding. One might be shapes. The children can divide shapes in to equal/unequal parts. Focus should be on the term '**equal parts**'. Children do not need to use the word 'congruent' at this stage but should understand the concept of the parts looking the same. The children need to be exposed to object that are the same but have been divided into unequal groups. E.g. 3 squares that have all been divided into 4 part but in different ways – as seen above.

Stage 3

Identifying, representing and comparing unit fractions

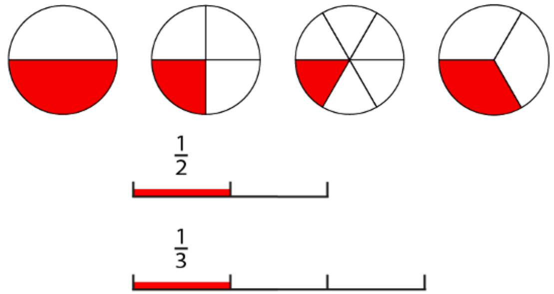
Example question: $\frac{1}{3} > \frac{1}{4} > \frac{1}{10} > \frac{1}{1000}$

Concrete:



At this stage, the children should be looking at the same model alongside each other and explore how the same whole can be divided into different numbers of equal parts. '**What's the same, what's different?**' It should be made clear to the children that a 'whole' can be whatever we define it. The children should also be exposed to situations and activities where the parts do not look the same, but the quantity within each part is equal. The children in the class could be the concrete resource, the class being the whole and the children being the parts.

Pictorial:



Give the children plenty of practice looking at shapes, lines and sets, and identifying how many equal parts each is divided into. Make sure that the language used in the previous stages is also used here (part, whole, equal parts, numerator, denominator, etc.).

Abstract:

$$\frac{1}{3} > \frac{1}{4} > \frac{1}{10} > \frac{1}{1000}$$

The children should be taught to write fractions using the **'bar first'** method. This is where the children draw the division bar first before writing in the digits. **They should be drawing the bar, then the denominator and then the numerator.** This supports children's understanding of a fraction being part of a whole that has been divided.

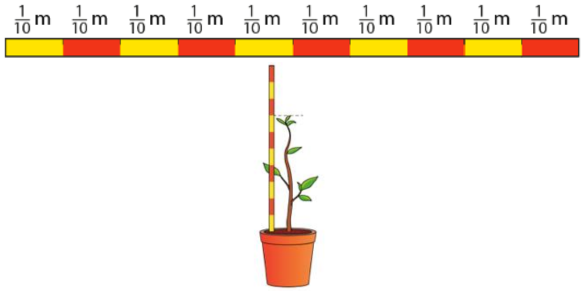
When working in the abstract at this stage, the children need to understand, **'when the numerator is the same, the larger the denominator the smaller the fraction'**.

Stage 4

Identifying, representing and comparing non-unit fractions

Example question: $\frac{4}{5} > \frac{3}{5} > \frac{2}{5}$

Concrete:

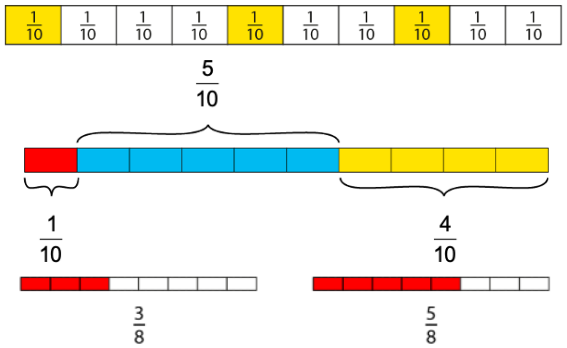


The whole metre is divided into __ equal parts.

The plant measures — of the whole metre.

Non-unit fractions are introduced through their connection to unit fractions. They are simply 'multiples' of unit fractions. These connections need to be made early on to support their understanding of non-unit fractions. While non-unit fractions are first being introduced, the language **'five one-sixths'** should be used in the place of five sixths. Meter sticks are a good concrete resource to use to demonstrate tens. Concrete representations should allow children to explore fractions of a whole and what it means to have more than one part of it – non unit fractions.

Pictorial:



When the meter ruler is used as a concrete resource, it can easily be transferred into pictorial bar models. The children should be exposed to area models, linear models and as quantities.

When using pictorial resources, it should be clear whether the parts are equal or unequal parts.

Abstract:

$\frac{4}{6}$ is four lots of $\frac{1}{6}$

$\frac{4}{5}$ is four lots of $\frac{1}{5}$

I know that $\frac{1}{6}$ is less than $\frac{1}{5}$

So, four lots of $\frac{1}{6}$ is less than four lots of $\frac{1}{5}$

$\frac{1}{4}$ is one lot of $\frac{1}{4}$

$\frac{3}{4}$ is three lots of $\frac{1}{4}$

I know that 1 is less than 3, so $\frac{1}{4}$ is less than $\frac{3}{4}$

As in the previous stage, the children should be using the 'bar first' method for writing fractions. The abstract should still be being used alongside the concrete and pictorial as this will help children understand the concept of non-unit fractions.

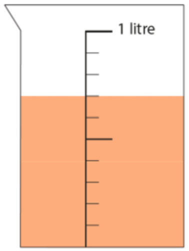
When working in the abstract at this stage, the children need to understand, **'when the denominator is the same, the smaller the numerator the smaller the fraction'**.

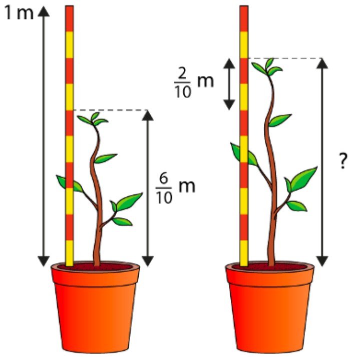
Stage 5

Adding and subtracting fractions within one whole

Example question: $\frac{3}{9}+\frac{4}{9}=\frac{7}{9}$ **and** $\frac{8}{9}+\frac{3}{9}=\frac{5}{9}$

Concrete:

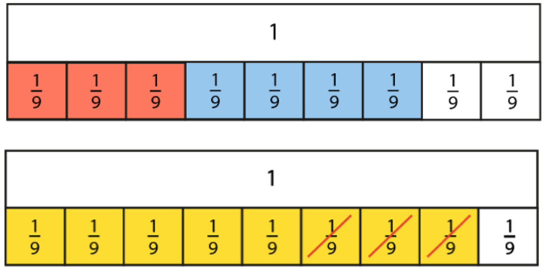




$\frac{7}{10} - \frac{3}{10} = \frac{4}{10}$

At this stage, it is important that the children understand that non-unit fractions are the repeated addition of unit fractions. This helps them to not add or subtract the denominators when adding and subtracting fractions. Alongside concrete resources, using stories and real-life situations should be used to support the children's understanding of fractions. For example, Jack eats three ninths of a pizza, Beth eats two ninths of a pizza. How much pizza did they eat altogether.

Pictorial:



Pictorial representations should be used heavily at this stage. They should be used both as questions and as a way for children to explain their reasoning. Again, stories and real-life problems are key to the children's understanding of adding and subtracting fractions. **Number lines** and **bar models** are one of the most beneficial pictorial representations that can be used.

Generaliser –
'When adding or subtracting fractions with the same denominators, just add or subtract the numerators.'

Abstract:

$\frac{3}{9}$ is 3 lots of $\frac{1}{9}$

$\frac{4}{9}$ is 4 lots of $\frac{1}{9}$

I know that 3 + 4 = 7

So I know that $\frac{3}{9}+\frac{4}{9}=\frac{7}{9}$

$\frac{8}{9}$ is 8 lots of $\frac{1}{9}$

$\frac{3}{9}$ is 3 lots of $\frac{1}{9}$

8 - 3 = 5

So $\frac{8}{9} - \frac{3}{9}$ is $\frac{5}{9}$

At this stage the written abstract should be left until later in the sequence of lessons. This is the help prevent the children from slipping into the misconceptions that they need to add/subtract the denominator as well as the numerator. Once the children understand adding and subtracting fractions with the same denominator, they should also be taught about the inverse and how it works in the same way as whole numbers.

3 + 4 = 7   7 – 4 = 3

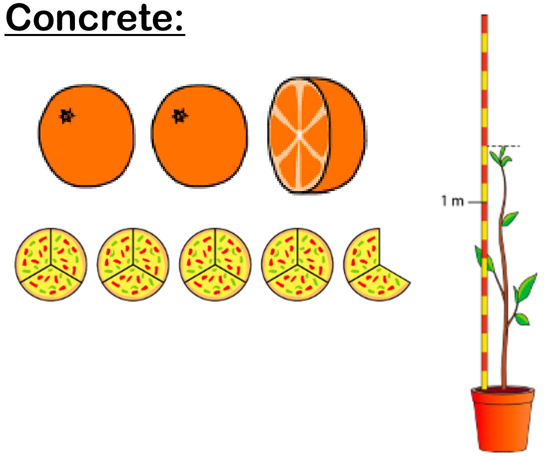
$\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$ $\frac{7}{10}-\frac{4}{10}=\frac{3}{10}$

Stage 6

Improper and mixed number fractions – crossing the whole

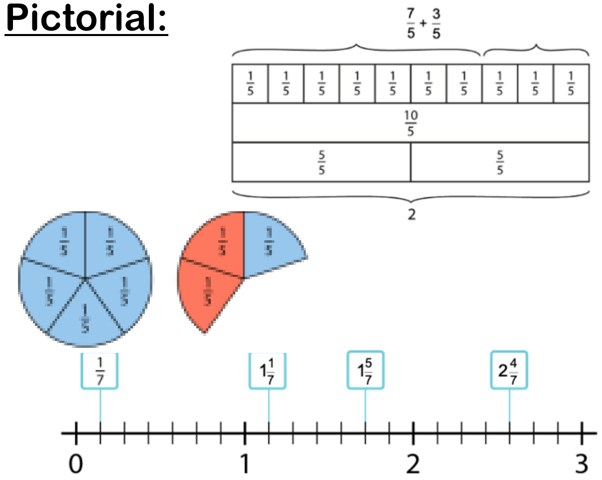
Example question: $\frac{7}{5}+\frac{3}{5}=\frac{10}{5}=2$

Concrete:



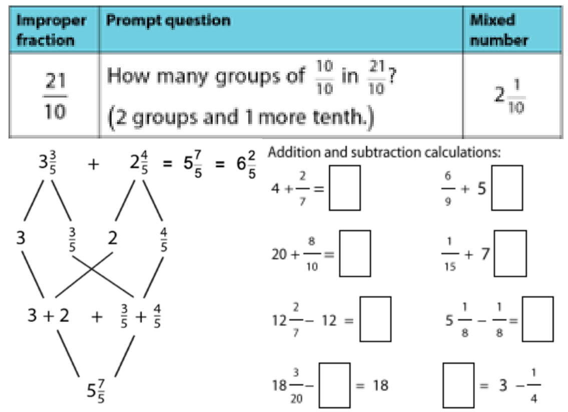
This will be the first time that the children experience fractions crossing the whole. They will do this initially through mixed number fractions and then move on to improper fractions. Concrete manipulatives are critical in ensuring the pictorial area models can be understood and accessed by all; fraction tiles can be useful for this.

Pictorial:



The area models used at this stage should be mostly in a linear form (similar to a bar model) or in a circular form (similar to a pie chart). It is important that children are exposed to both forms. The linear model is easier for children to draw but does not clearly show when fractional parts exceed a whole. Number lines should be shown alongside area models as often as possible. The part-part-whole model is useful for encouraging children to connect back to previous addition and subtraction.

Abstract:



When moving into the abstract, children should be exposed to questions that both have the whole number as the first addend and some with the fraction as the first addend. Units of measure are a good way of presenting abstract questions to the children to give them context and support understanding.

Stage 7

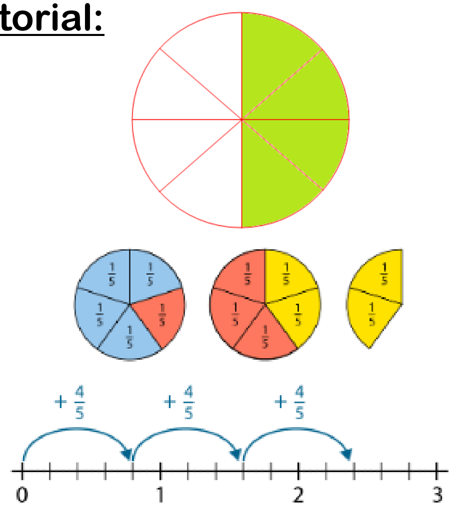
Multiplying whole numbers and fractions

Example question: $\frac{1}{8} \times 4 = \frac{4}{8}$ **or** $\frac{4}{5} \times 3 = \frac{12}{5} = 2\frac{2}{5}$

Concrete:

When children are first introduced to this concept, the teacher should clearly reference multiplication of whole number that they have learnt previously. This means that is should be presented to them as repeated addition – as seen previously in the multiplication section. Concrete resources should be used alongside the pictorial and abstract resources.

Pictorial:



Number lines are one of the key pictorial resources when representing repeated addition. They are also useful for showing when multiplying they will often cross the whole. Area models, number lines and bar models should be used heavily at this stage.

Abstract:

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$4 \times \frac{1}{8}$$

$$\frac{1}{8} \times 4$$

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 4 \times \frac{1}{8} = \frac{1}{8} \times 4$$

$$3 \times \frac{4}{5} = \frac{4}{5} + \frac{4}{5} + \frac{4}{5}$$

$$3 \times \frac{4}{5} = \frac{12}{5} = 2\frac{2}{5}$$

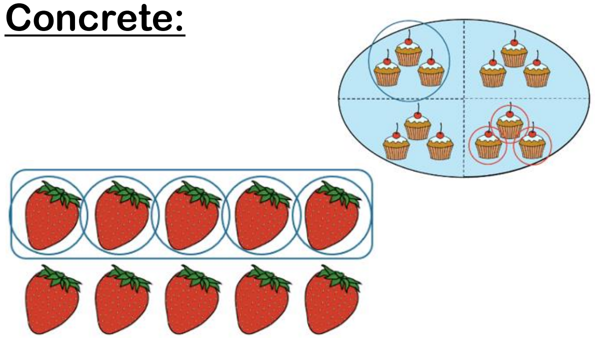
When working in the abstract, the children will either continue to use **repeated addition** or work with **scaling –** finding the value of the unit fraction and then multiplying it by the number of them you need (the numerator). E.g. $\frac{2}{3}$ x 60. If $\frac{1}{3}$ of 60 is 20 then $\frac{2}{3}$ of 60 is 40.

Stage 8

Finding equivalent fractions and simplifying fractions

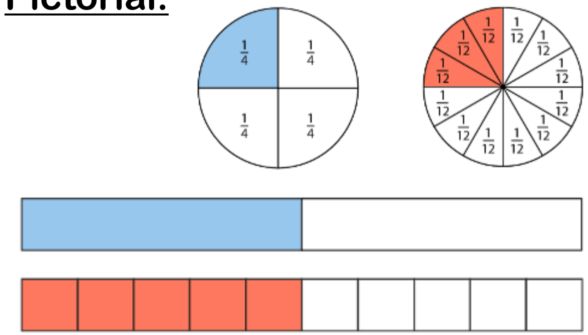
Example question: $\frac{1}{4} = \frac{3}{12}$

Concrete:



Fractions are essentially multiplicative comparisons between a part (expressed by the numerator) and a whole (expressed by the denominator). The children will need to understand that it is the sizes of these, relative to each other, that are critical in determining the value of a fraction. When simplifying fractions, the scale factor that the fraction has been changed by should be the same for both the numerator and denominator. At this stage, practical (concrete) activities, such as pouring liquids or rice, will support the children's fraction sense. Through this, children can see how pairs of fractions represent the same part of a whole.

Pictorial:



Placing fractions on number lines and seeing that equivalent fractions all sit at exactly the same position will help address any misconceptions that fractions with larger digits have a larger value. Circular area models should be reduced in this section and linear area models and number lines should be used in their place as they support the transition into the abstract representations. When models and diagrams are used the '=' symbol should not be used to compare them. This is because it is not always clear to the children that it is the fraction that is being compared. This should be saved for the abstract, where it is clear that they are comparing fractions.

Abstract:

$$\frac{1}{2} = \frac{5}{10} \qquad \frac{1}{4} = \frac{3}{12}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}$$

$$\frac{8}{12} \overset{\div 2}{\underset{\div 2}{=}} \frac{4}{6} \qquad \frac{4}{6} \overset{\div 2}{\underset{\div 2}{=}} \frac{2}{3} \qquad \frac{8}{12} \overset{\div 4}{\underset{\div 4}{=}} \frac{2}{3}$$

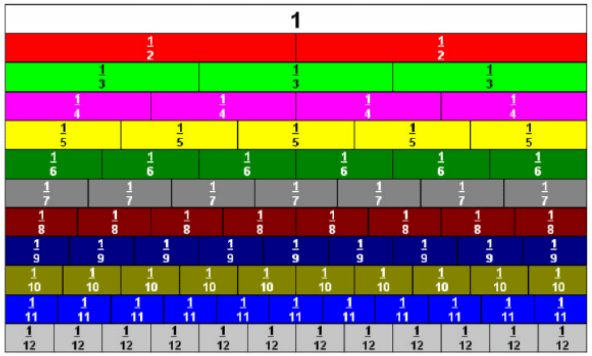
The idea that when we find an equivalent fraction, we are multiplying or dividing the numerator and denominator by the same amount, but we are not multiplying the whole fraction itself, can be quite challenging for children to understand. When working in the abstract, it is important that children understand the fundamental differences between the two concepts. Instead of talking about multiplying and dividing, instead use 'increase/decrease by the same scale factor'.

Stage 9

Common denomination: + and - by finding a common denominator

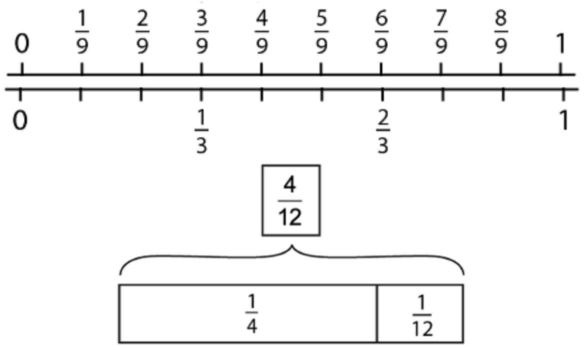
Example question: $\frac{1}{3}+\frac{1}{12}=\frac{5}{12}$ **becomes** $\frac{4}{12}+\frac{1}{12}=\frac{5}{12}$

Concrete:



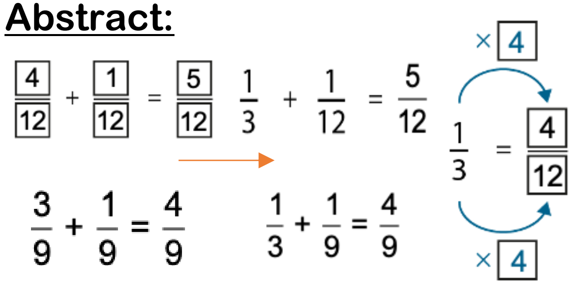
Fractions that are being added or subtracted are considered as either related (one denominator is a multiple of the other) or non-related (neither denominator is a multiple of the other). Using a fraction wall can support children in their understanding of related fractions. Having a times table chart can also support both addition and subtraction of both related and non-related fractions. Once the children can convert the question so that the fractions have the same denominator, they need to apply their knowledge of adding and subtraction fractions with the same denominator from stage 5.

Pictorial:



Teaching at this stage should be heavily supported by diagrams, such as number lines and bar charts, but the level of scaffolding should be greatly reduced to encourage them to develop fluency once they have secured conceptual understanding. When placing fractions on a number line, the children are able to see how the fractions in the question relate to one another. Bar charts give the children a sense of the size of the fractions in comparison to one another.

Abstract:



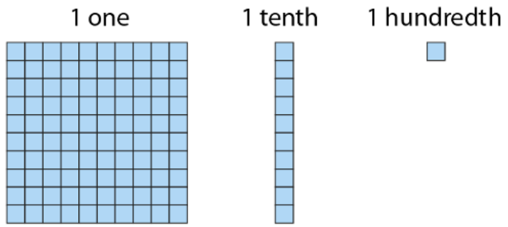
Try to balance the move to more efficient procedures with sense-checking their calculations. For example, $\frac{1}{3}+\frac{1}{4}$ can't be $\frac{1}{7}$ because $\frac{1}{7}$ is smaller than both the addends. When working in the abstract, if the children are not secure in their times tables, ensure that they have a times table grid next to them.

Stage 10

Multiplying fractions and dividing fractions by a whole number

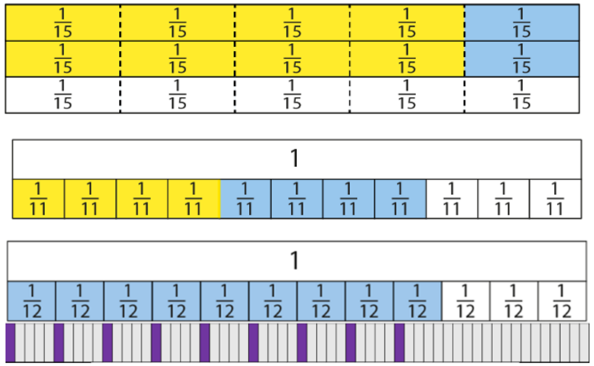
Example question:

Concrete:



A fundamental concept is that multiplying a number by a proper fraction makes it smaller. This conceptual understanding is just as important as being able to perform a procedure for multiplying and dividing fractions. The idea that when we multiply by a proper fraction, we are making a number smaller, is really significant because in some branches of higher mathematics, division ceases to be a concept that is used, since any division can be replaced by multiplication. For example, dividing by 4 becomes multiplying by $\frac{1}{4}$.

Pictorial:



As the procedures for multiplying pairs of fractions and dividing fractions by whole numbers are so simple children may become fluent quickly. Initially, if the children have picked up the procedure quickly, they should be able to draw diagrams to explain the process.

Abstract:

$$\frac{4}{5} \times \frac{2}{3} = \frac{8}{\boxed{15}} \qquad \frac{2}{3} \times \frac{4}{5} = \frac{8}{\boxed{15}}$$

$$\frac{8}{11} \div 2 = \frac{4}{11}$$

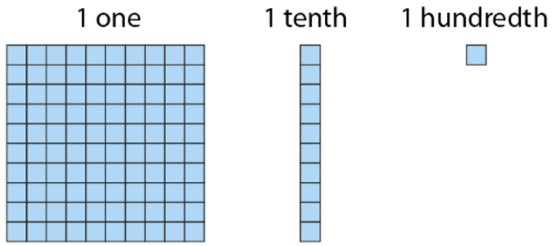
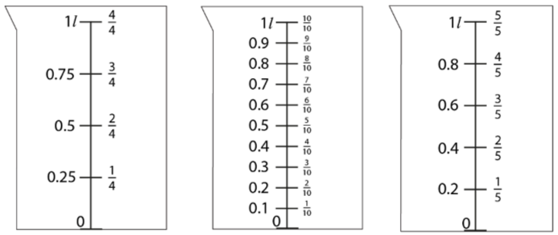
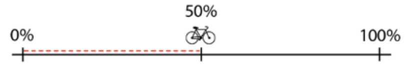
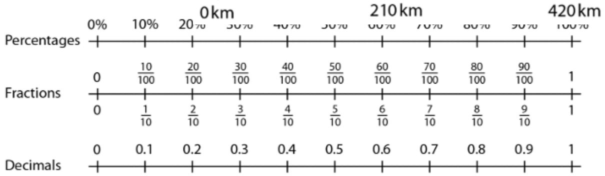
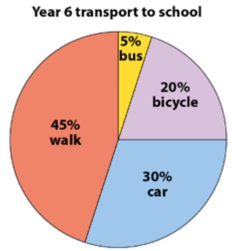
$$\frac{9}{12} \div 5 = \frac{9}{12} \times \frac{1}{5} = \frac{9}{60}$$

If is useful for children to simplify fractions before completing these types of calculations. The children should also be using their sense-checking – making sure the answer is smaller than the number in the question. This demonstrates their deeper understanding. Questions should be written with the multiplier and multiplicand in either position. E.g. three lots of $\frac{1}{5}$ can be written as 3 x $\frac{1}{5}$ or $\frac{1}{5}$ x 3 or $\frac{1}{5}$ of 3. With division, it would be $\frac{1}{2}$ x $\frac{1}{3}$ or $\frac{1}{2}$ ÷ 3.

Stage 11

Linking fractions, decimals and percentages

Example question:

Concrete:	**Pictorial:**	**Abstract:**
 	  	$80\% = \frac{80}{100} = \frac{4}{5}$ $45\% = \frac{45}{100} = \frac{9}{20}$ $12\% = \frac{12}{100} \underset{\div 4}{\overset{\div 4}{=}} \frac{3}{25}$ 10% $\frac{1}{10}$ 0.1 20% $\frac{2}{10}$ 0.2 30% $\frac{3}{10}$ 0.3 40% $\frac{4}{10}$ 0.4 50% $\frac{5}{10}$ 0.5 60% $\frac{6}{10}$ 0.6 70% $\frac{7}{10}$ 0.7 80% $\frac{8}{10}$ 0.8 90% $\frac{9}{10}$ 0.9 100% $\frac{10}{10}$ 1
At this stage, the children should be meeting percentages for the first time, meaning they should already have a secure understanding of both fractions and decimals. As this is the first-time children will be looking at percentages, sufficient time must be spent discretely looking at percentages. When using concrete resources Base 10 is an easy resource for the children to use as the 100 square can be used as the whole/100%.	Once the children have a secure understanding of percentages, they should then start looking at equivalents. This should be done with concrete and pictorial representations interchangeably. Key pictorial representations to use are number lines, as the children can directly see the equivalents side by side.	When first moving to the abstract, the children should have pictorial or concrete (or both) representations to support their understanding of percentages and how they link to fractions and decimals.